Improved Quantum Circuits with Qudits and Dirty Ancillae

PRANAV GOKHALE, JONATHAN M. BAKER, and FREDERIC T. CHONG, University of Chicago, USA

Architectural research has made significant contributions to the prospects for near-term quantum computation by leveraging techniques such as mapping, scheduling, and parallelism. In this work, we systematically explore two additional techniques that can reduce space (ancillae) and time (circuit depth) requirements by accessing computational resources that are traditionally ignored. The first technique is the use of d-level qu*dits* in favor of the conventional two-level qu*bit* (quantum bit) abstraction. The second technique is the exploitation of dirty ancillae: idle qubits (or qudits) from other parts of computation which can be borrowed with the promise that they will be restored to their original states. We introduce new results for more efficient quantum circuits, particularly for multiply-controlled unitary gates, which are critical for many quantum algorithms. Our ongoing work includes the development of an error model that accounts for the cost of using qudits. We are also building a software framework for synthesizing logic circuits that exploit qudits and dirty ancillae, with the goal of maximizing the amount of useful computation possible.

Recent advances in both hardware and software for quantum computation have demonstrated significant progress towards practical outcomes. While earlier efforts focused on longer-term systems employing full error correction to execute algorithms like Shor's [10] and Grover's [3], recent work has focused on NISQ (Noisy Intermediary-Scale Quantum) computation [8]. The NISQ regime considers near-term machines with just tens of qubits, limited connectivity between qubits, modest coherence times, and modest gate speeds.

In this regime, computation is severely resource constrained, so fully optimizing quantum operations is critical for successful computation. Prior architectural research has explored techniques such as mapping, scheduling, and parallelism [1, 6] to extend the amount of useful computation possible. In this work, we systematically consider two techniques: qudits and dirty ancillae. The two techniques are interrelated in that both exploit computation spaces that are readily available in quantum systems but are traditionally ignored.

The first technique involves breaking the two-level qu*bit* (quantum bit) abstraction in favor of d-level qu*dits*. Unlike classical computers, which operate in binary states at the physical level, quantum computers have natural access to an infinite spectrum of discrete energy levels. In fact, hardware must actively suppress the higher level states in order to achieve the two-level qubit approximation. Hence, quantum systems are well matched to d-level qudits. Prior experimental work [7, 9] has succeeded in using qu*trits* to reduce the circuit depth of the Toffoli gate, an important primitive in many quantum algorithms. Our methodology maintains input and output states as two-level qubits, and only occupies higher level qudit states during the computation. We are developing an error model that accounts for the cost of temporarily occupying the higher level qudit states.

The second technique involves dirty ancillae. The core idea is that qubits which are idle at a particular timestep may be used as ancillae by adjacent computations, provided that they are restored to their original unknown ("dirty") state after use. Dirty qubits have been recently exploited to yield resource savings in arithmetic circuits for important quantum algorithms [2, 4].

The primary insight of our work is that the composition of these two techniques leads to novel circuit designs that have asymptotically smaller circuit depths, as opposed to the constant ($\log_2 d$) reduction that is achieved solely from using qudits. A key contribution of our work is a circuit design that uses dirty qutrits to perform an n-controlled gate (i.e. gate is applied only when all of the n controls are ON) with logarithmic circuit depth and no additional ancillae. The circuit is demonstrated for $n = 12$ in Figure 1. Previous designs for logarithmic-depth n-controlled gates have

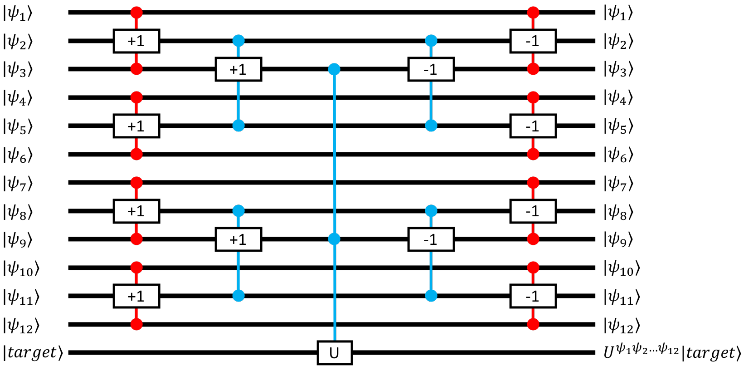

Fig. 1. Circuit for 12-controlled U gate using dirty qutrits. The +1 gate operates on qutrit states as $+1\,|i\rangle = |i+1 \mod 3\rangle$. The red controls activate for $|1\rangle$ and the blue controls activate for $|2\rangle$.

required linear ancillae [5]. Since the n-controlled gate is a subcircuit of the Grover's algorithm, our design improves the circuit depth of Grover search.

We also propose a circuit that implements a $+k$ adder (for any compile-time constant k) with linear depth and no ancillae, which is an improvement over previous work [2, 4]. This adder is a critical subcircuit for implementing Shor's algorithm.

In addition to modeling errors for qudits, we have ongoing work towards a software toolkit that performs circuit synthesis based on our new circuit constructions. We plan to extend existing tools for reversible circuit design such as RevKit [11]. We expect to apply this tool across a range of quantum algorithms and error models to quantify the advantage of using dirty qudits.

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